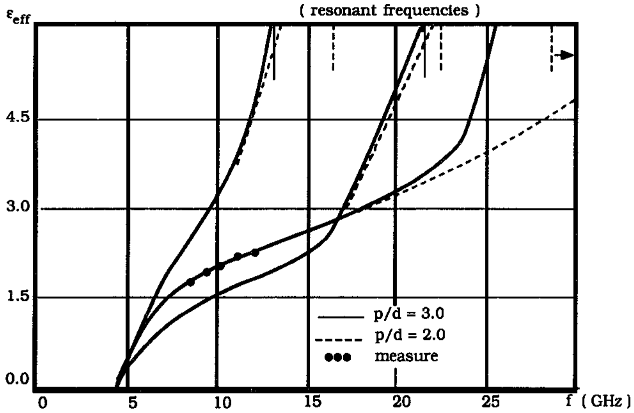

Fig. 2. Dispersion characteristics of periodic finlines with arbitrarily located stubs: $w_1 = 0.5$ mm, $w_2 = 4.5$ mm, $s_1 = 4.83$ mm, $p = 3$ mm.

On the other hand, by moving the stubs from $s_2 = 2.83$ mm (symmetric case) to $s_2 = 4.83$ mm (offset case) the resonant frequency goes down considerably. Another interesting phenomenon is that the resonant frequency can effectively be changed by adjusting the period length without varying the dispersion characteristics over the passband range unless the frequency is in the shadow of resonance.

The resonance phenomenon arises in two cases:

$$s_1 - s_2 = C(2k-1)\lambda/4$$

and/or

$$w_2 - w_1 - s_1 = C(2k-1)\lambda/4$$
$$p = n\lambda/2 \qquad (k, n = 1,2,3,\cdots).$$

The coefficient C is determined by geometric conditions. It can easily be seen that the passband and stopband will occur periodically with the frequency.

V. Conclusion

A new concept called modal spectrum in the propagation direction has been introduced and successfully applied in the theoretical analysis. It makes possible the direct use of the three-dimensional spectral-domain approach in both symmetrically and asymmetrically loaded periodic structures. Several examples based on this unified algorithm illustrate the slow-wave phenomenon as well as passband and stopband behavior related to the cutoff and resonant frequencies. The dielectric losses can be involved.

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On the Calculation of Conductor Loss on Planar Transmission Lines Assuming Zero Strip Thickness

Peter Heitkämper and Wolfgang Heinrich

Abstract —**The incompatibility of the zero-strip-thickness assumption with conductor loss calculation based on the common perturbation approach is addressed. Numerical results are shown that demonstrate the unbounded behaviour of the attenuation constant in this case. This observation is of specific interest because it applies to various data on loss given in the literature.**

I. The Problem

Conductor loss on planar transmission lines such as microstrip, coplanar waveguide, and slotline is usually calculated by means of a perturbation approach. One starts from an analysis of the lossless waveguide and then determines the attenuation from the corresponding surface currents on the conductors. Assuming the tangential magnetic field to remain approximately unchanged by the losses, one arrives at the well-known formula

$$P_c = \frac{1}{2} R_s \int_C |\vec{H}_t|^2 \cdot ds \qquad (1)$$

where P_c denotes the dissipated power per unit length, R_s the surface resistance of the conductors, $\vec{H}_t$ the tangential magnetic field, and C the integration path along the contour of the conductors. Consequently, for the attenuation constant α_c caused by the conductor losses, one has

$$\alpha_c = \frac{1}{2} \cdot \frac{P_c}{P_z} \qquad (2)$$

with P_z being the total power transported in the longitudinal direction along the waveguide. Clearly, such a procedure makes

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P. Heitkämper is with the Institut für Netzwerk- und Signaltheorie, Technische Hochschule Darmstadt, Merckstrasse 25, D-6100 Darmstadt, Germany.

W. Heinrich is with the Institut für Hochfrequenztechnik, Technische Hochschule Darmstadt, Merckstrasse 25, D-6100 Darmstadt, Germany.

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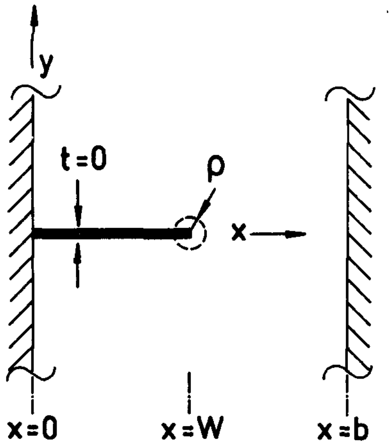

Fig. 1. The structure under consideration. Edge of a perfectly conducting strip with zero thickness.

sense only if the fields in the lossy case do not differ significantly from those of the lossless structure.

Because of its effectiveness, the spectral-domain approach (SDA) assuming zero strip thickness ($t=0$) is commonly used when analyzing the lossless waveguide (see, for instance, [1] and [2] and, more recently, [3]). The fields derived by this analysis are then used to calculate α_c according to (1) and (2). At this point, one encounters the problem that was pointed out by Pregla [4]: It is well known that the tangential magnetic field, H_t, at the edge of an infinitely thin and perfectly conducting strip (see Fig. 1) behaves according to

$$H_t \sim \frac{1}{\sqrt{\rho}} \qquad \text{for } \rho \to 0 \tag{3}$$

with ρ denoting the radial distance from the edge. Inserting (3) into (1) then leads to an integrand $\sim \rho^{-1}$. Consequently, the integral in (1) diverges:

$$\int \rho^{-1} \cdot d\rho = \ln \rho \quad \text{with } |\ln \rho| \to \infty \qquad \text{for } \rho \to 0. \tag{4}$$

In practice, however, this problem does not become apparent. In the numerical calculations, the infinite series representing the fields above and below the strip are truncated in principle, because only a finite number M of spectral expansion terms can be taken into account. This holds even if unbounded basis functions for the strip currents are employed.

II. Numerical Considerations and Results

Regarding the common spectral-domain approach, the truncation of the infinite sum corresponds to a minimum spatial resolution s_x in the x direction [2]. Let us consider the geometry shown in Fig. 1. The fields are expanded in sine and cosine terms with the separation constant $k_{xm} = m\pi/b$, e.g.,

$$E_x \sim \sum_{m=0}^{M} K_m \cdot \cos\left(\frac{m\pi}{b} \cdot x\right). \tag{5}$$

Hence one obtains a resolution $s_x = b/M$. Therefore, limiting the number of eigenfunctions to M means that one can approach the singularity at $x = W$ merely within this spatial resolution (see [2]). Equally, the calculation of the dissipated power P_c according to (1) is affected: Instead of the integral

$$I = \int_{x=0}^{x=W} |\vec{H}_t|^2 \cdot dx \tag{6}$$

one evaluates

$$I' = \int_{x=0}^{x=W-b/M} |\vec{H}_t|^2 \cdot dx. \tag{7}$$

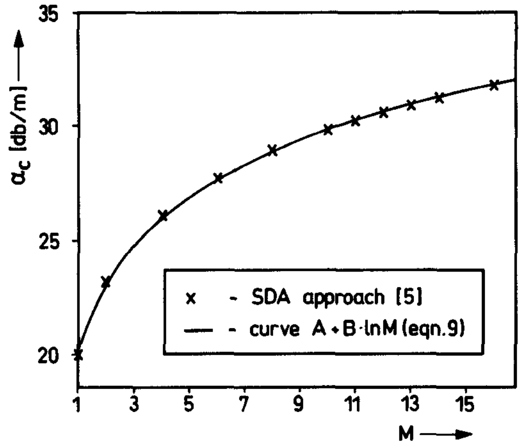

Fig. 2. Attenuation α_c due to conductor loss against the number of spectral terms M (30 μm wide microstrip on GaAs at 4 GHz). Results obtained by SDA [5] (crosses) and fitted logarithmic function $\alpha = 20.1256 + 4.21355 \cdot \ln M$ (solid line), respectively.

This integral, however, may be solved analytically. One obtains

$$I' \sim \ln W + \ln \frac{b}{M} \tag{8}$$

and thus

$$I' = A + B \cdot \ln M \tag{9}$$

with A and B being constants. As can be seen easily from this equation, the attenuation α_c obtained does *not* converge with increasing number M of spatial eigenfunctions because $\ln M$ is unbounded.

The following numerical results support this observation. In Fig. 2 the values of α_c are drawn as a function of the spectral truncation index M. The common shielded microstrip structure was studied [5] using the spectral-domain technique assuming $t=0$ and the perturbation approach according to eqn. (1) and (2). Additionally, a logarithmic function $A + B \ln M$ is depicted, with the coefficients A and B being fitted to the SDA results. The logarithmic increase of α_c with M is confirmed very clearly.

In conclusion, no convergence can be postulated for the attenuation constant calculated from the lossless case assuming $t=0$. Moreover, an arbitrarily large value of α_c can be achieved by increasing M, where the limit is ultimately determined by the computer capabilities available. Particularly, there are no reliable criteria available for how M should be chosen in order to obtain realistic results for the case where $t>0$. Note that the logarithmic increase seems to indicate a saturation behaviour, but as we know from (9) no saturation takes place.

Nevertheless, the results obtained by the approach described above agree with experiment fairly well in several cases. This observation may be explained by the relatively slight increase of the "ln" curve and by the poor accuracy when measuring low-loss planar transmission lines.

Additionally, one consideration is suitable in explaining the problem when treating lines of zero strip thickness as lossy ones. For thin strips of high conductivity one may take, in sequence, the two limits $\kappa \to \infty$ and $t \to 0$. The conductivity κ must be chosen high enough, however, so that the corresponding skin depth δ is always small compared with the strip thickness t. Hence there exists a lower bound restriction for κ that depends on t. Owing to this interdependence, the order of the two operations is fixed. It is not possible to interchange the two limiting procedures as done when setting $t=0$ first and then calculating the losses due to a finite κ. In mathematical notation, this reads

$$\lim_{t \to 0}\left[\lim_{\kappa \to \infty}\Big|_{\delta(\kappa) \ll t} \cdots\right] \neq \lim_{\kappa \to \infty}\left[\lim_{t \to 0} \cdots\right].$$

III. Conclusions

When calculating the conductor loss of planar transmission lines by means of perturbation methods, the assumption of zero strip thickness becomes critical: the surface current integral at the strip edges does not exist. In the numerical analysis, the results for the attenuation constant do not converge. For an increasing number M of spectral eigenfunctions, α_c approaches infinity. However, α_c shows a logarithmic dependence on M so that, in practice, the unbounded behavior may be easily overlooked or misinterpreted. As a consequence, attenuation results obtained on the basis of zero-strip-thickness approaches, such as the common spectral-domain technique, should be handled very critically and checked against measurements.

Acknowledgment

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A Closed-Form Spatial Green's Function for the Thick Microstrip Substrate

Y. L. Chow, J. J. Yang, D. G. Fang, and G. E. Howard

***Abstract*—The spatial Green's function for the open microstrip structure, especially with a thick substrate, is generally represented by time-consuming Sommerfeld integrals. In this paper, through the Sommerfeld identity, a closed-form spatial Green's function of a few terms is found from the quasi-dynamic images, the complex images, and the surface waves. With the numerical integration of the Sommerfeld integrals thus avoided, this closed-form Green's function is computationally very efficient. Numerical examples show that the closed-form Green's function gives less than 1% error for all substrates and source-to-field distances.**

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Y. L. Chow, J. J. Yang, and G. E. Howard are with the Department of Electrical and Computer Engineering, University of Waterloo, Waterloo, Ont., Canada N2L 3G1.

D. G. Fang is with the East China Institute of Technology, Nanjing, China 210014.

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I. Introduction

In the modeling of microwave integrated circuits (MIC's) and microstrip antennas, much effort has to be dedicated to the computation of Sommerfeld integrals. For a thin microstrip substrate, say $h/\lambda_\epsilon < 0.05$, the quasi-dynamic image model intuitively developed by Chow [1] is a good replacement for the Sommerfeld integrals. For a thick substrate and when the distance from the source point to the field point is great, however, this replacement deteriorates because of its neglect of the surface and leaky wave effects. To accurately model the thick-substrate microstrip circuits, it appears that the time-consuming numerical integration of Sommerfeld integrals has to be performed [2], [3]. Although the exact image method was developed for the microstrip structure [4], where the Sommerfeld integrals were replaced by certain alternative infinite integrals, it was shown in [5] that this alternative type of numerical integration is still rather time-consuming.

In this paper, a closed-form Green's function for a thick microstrip substrate is presented. This Green's function consists of three parts: $G = A + B + C$, where A represents the contribution from a few *quasi-dynamic images* dominating in the near-field region, C represents the contribution from *surface waves* dominating in the far-field region of the substrate surface, and B represents the contribution from the *complex images*, which are related to leaky waves and are very important in the intermediate field region. With this closed form, numerical integration of Sommerfeld integrals is completely avoided. It will be shown below numerically that at any frequency, this closed-form Green's function gives less than 1% error compared with the numerical integration of Sommerfeld integrals in the whole range of substrate surfaces.

II. Theory

Consider an x-directed electric dipole of unit strength located above a microstrip substrate, as shown in Fig. 1. The spectral-domain potentials in the air region can be represented as follows:

$$\tilde{G}_A^{xx} = \frac{\mu_0}{4\pi}\frac{1}{j2k_{z0}}\left[e^{-jk_{z0}(z-z')} + R_{\mathrm{TE}}e^{-jk_{z0}(z+z')}\right] \tag{1a}$$

$$\tilde{G}_q = \frac{1}{4\pi\epsilon_0}\frac{1}{j2k_{z0}}\left[e^{-jk_{z0}(z-z')} + \left(R_{\mathrm{TE}} + R_q\right)e^{-jk_{z0}(z+z')}\right] \tag{1b}$$

where

$$R_{\mathrm{TE}} = -\frac{r_{10}^{\mathrm{TE}} + e^{-j2k_{z1}h}}{1 + r_{10}^{\mathrm{TE}}e^{-j2k_{z1}h}} \tag{2a}$$

$$R_q = \frac{2k_{z0}^2(1-\epsilon_r)(1-e^{-j4k_{z1}h})}{(k_{z1}+k_{z0})(k_{z1}+\epsilon_r k_{z0})\left(1 + r_{10}^{\mathrm{TE}}e^{-j2k_{z1}h}\right)\left(1 - r_{10}^{\mathrm{TM}}e^{-j2k_{z1}h}\right)} \tag{2b}$$

$$r_{10}^{\mathrm{TE}} = \frac{k_{z1} - k_{z0}}{k_{z1} + k_{z0}} \qquad r_{10}^{\mathrm{TM}} = \frac{k_{z1} - \epsilon_r k_{z0}}{k_{z1} + \epsilon_r k_{z0}} \tag{3}$$

$$k_{z0}^2 + k_\rho^2 = k_0^2 \qquad k_{z1}^2 + k_\rho^2 = \epsilon_r k_0^2. \tag{4}$$

In (1), $\tilde{G}_A^{xx}$ stands for the x component of spectral-domain vector potential $\vec{A}$ created by the x-directed electric dipole, and $\tilde{G}_q$ stands for the spectral-domain scalar potential associated with one charge of the dipole. R_{TE} and R_q take into account the effects of the microstrip substrate. The spectral-domain Green's functions of (1) were given in a more compact form by

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